Acta Simulatio - International Scientific Journal about Simulation
Volume: 11 2025 Issue: 4 Pages: 27-31 ISSN 1339-9640

https://doi.org/10.22306/asim.v11i4.127

Received: 12 Sep. 2025
Final revised: 21 Nov. 2025
Accepted: 15 Dec. 2025

Estimation of cumulative distribution for noncentral distributions

Prasanth Sambaraju
Independent Researcher, Hyderabad, India, prashanth.kng1@gmail.com

Keywords: noncentral, cumulative distribution function, hypothesis testing, Type II error, Power of t-test.

Abstract: Noncentral distributions are probability distributions that are used to model a random variable where a noncentrality parameter determines the location of this distribution. These distributions are used in statistics, especially in hypothesis testing to compute the power of a test. The manuscript aims to compute cumulative distribution for noncentral t distribution using built-in functions in Microsoft Excel. The results obtained were found to be comparable to reported values.

1 Introduction

The noncentral *t* distribution has two parameters, namely υ (degrees of freedom) and δ (noncentrality parameter), that exhibits a unimodal density. When the value of δ is zero, the distribution becomes Student's *t* distribution. Degrees of freedom is usually a positive integer, but when calculated using the Welch or Satterthwaite equation [1] it can be any positive value (1). If *X* and *Y* are independent random variables such that *X* is normally distributed N(δ,1) and *Y* follows χ^2 distribution with υ degrees of freedom, then the ratio follows a noncentral *t* distribution with δ as the noncentrality parameter having υ degrees of freedom.

$$\frac{X}{\sqrt{Y/\upsilon}} \tag{1}$$

The most important application of the noncentral *t* distribution is to compute of power of the *t* test [2]. Power of a *t*-test is probability that a *t*-test will correctly reject a false null hypothesis with mean of a normal population $N(\mu, \sigma^2)$. The test of the null hypothesis is H_0: $\mu \le \mu_0$ against the alternative H_A: $\mu > \mu_0$ based on a sample from this normal population, where the population mean μ is greater than μ_0 [3]. The noncentrality parameter is the normalized difference between μ_0 and μ [4] (2).

$$\delta = \frac{(\mu_0 - \mu)}{\left(\sigma/\sqrt{n}\right)} \tag{2}$$

The existing algorithms to compute the cumulative distribution function (cdf) involve different expansion and or recurrence [5]. The manuscript aims to use Microsoft Excel to compute cdf by using normal and chi-square distributions. This is achieved by generating random variates from the noncentral *t* distribution of specified δ and υ values as a ratio normal distribution of specified mean which, corresponds to υ and a standard deviation of 1 and a chi-square distribution with degrees of freedom corresponding to δ. Simulation using the data table function in Microsoft Excel was performed to obtain a more precise estimate of the cdf.

2 Methodology

2.1 Visualize noncentral distribution

To visualize noncentral *t* distribution random variates from the noncentral *t* distribution as described [6], the random variates were generated using built-in function in Microsoft Excel. To generate a noncentral *t* distribution with 6 degrees of freedom and a noncentrality parameter of 4, 10000 normal random numbers of mean 4 and standard deviation 1 and chi-square random numbers with 6 degrees of freedom obtained using NORM.INV and CHISQ.INV functions respectively. Then the by using Equation 1, noncentral *t* distribution random variates were obtained. A plot of the histogram of these values as shown in Figure 1 represents the density estimate of a noncentral *t* distribution with 6 degrees of freedom and a noncentrality parameter value of 4.

Acta Simulatio – International Scientific Journal about Simulation
Volume: 11 2025 Issue: 4 Pages: 27-31 ISSN 1339-9640

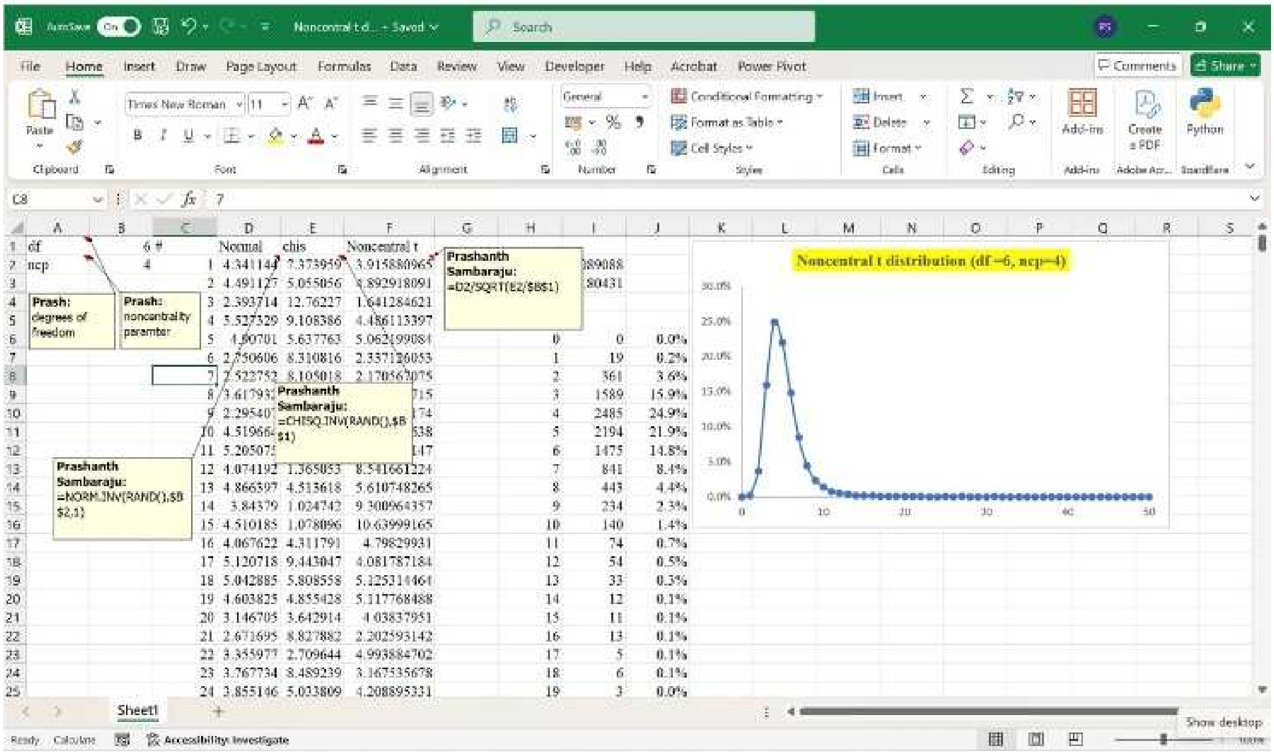

Figure 1 Computation of random variates of noncentral t distribution using Microsoft Excel. Histogram shown corresponds to noncentral t distribution density with 6 degrees of freedom and a noncentrality parameter value of 4

2.2 *Power of t test*

Power was determined for one sample *t* test using the data described here [7]. Simulation was performed using Table **1**. The Data Table can perform calculations at once without the need for any programming code. An introduction to the application of Data Table can be found here [9]. Power calculation for one sample *t* test using the Data Table function in Microsoft Excel [8]. The results are shown in

noncentral *t* distribution density plot is shown in Figure 2. Two sample t tests can be divided into two categories as shown in Figure 3.

Table 1 One sample t test power calculation

μ_0	850
μ	810
σ	50
N	19
δ	= (850-810)/(50/√19) = 3.487119
α	5%
degrees of freedom	= 19 – 1 = 18
critical *t* (Formula in Microsoft Excel)	= T.INV(1-5%/2,18) = 2.100922
Power [7]	0.909207
Power obtained from this method	0.909226 (after 2000 simulations)

Acta Simulatio - International Scientific Journal about Simulation
Volume: 11 2025 Issue: 4 Pages: 27-31 ISSN 1339-9640

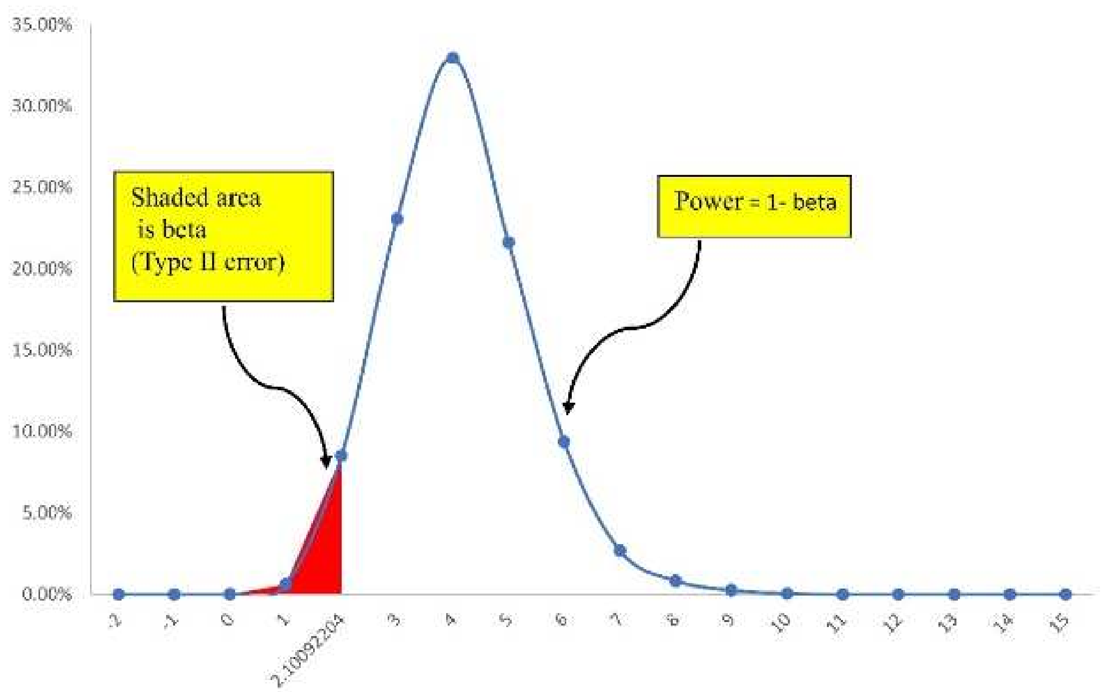

Figure 2 Power calculation using Microsoft Excel for example described [7]

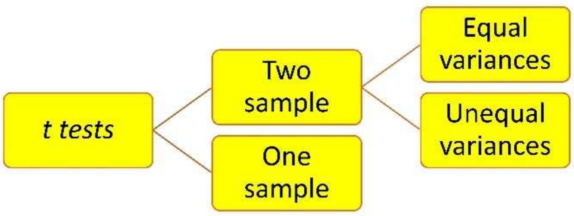

Figure 3 Different types of t tests

In case of one sample *t* tests and two sample *t* tests with equal variances, the degrees of freedom is always a positive integer. The noncentrality parameter for two sample *t* test with equal variances in given by (3)

$$\delta = \frac{(\mu_1 - \mu_2)}{\left(\sigma * \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)} \tag{3}$$

where (4)

$$\sigma = \sqrt{\frac{(n_1 - 1) * s_1^2 + (n_2 - 1) * s_2^2}{(n_1 + n_2 - 2)}} \tag{4}$$

σ is the pooled standard deviation
s_1 is the standard deviation for the first sample
s_2 is the standard deviation for the second sample
n_1 and n_2 are the sample size for first and second sample respectively.
and the degrees of freedom is given by [10] (5)

$$degress\ of\ freedom = n_1 + n_2 - 2 \tag{5}$$

For two sample *t* tests with unequal variances, the degrees of freedom can be approximated by Satterthwaite formula [1], which might result in fractional degrees of freedom (6).

$$v = \frac{\left(\frac{S_1{}^2}{N_1} + \frac{S_2{}^2}{N_2}\right)^2}{\frac{\left(\frac{S_1{}^2}{N_1}\right)^2}{N_1 - 1} + \frac{\left(\frac{S_2{}^2}{N_2}\right)^2}{N_2 - 1}} \tag{6}$$

where,
S_1 is the standard deviation for the first sample
S_2 is the standard deviation for the second sample
N_1 and N_2 are the sample size for first and second samples respectively.

The noncentrality parameter is given by (7)

$$\delta = \frac{(\mu_1 - \mu_2)}{\left(\sqrt{\frac{S_1}{N_1} + \frac{S_2}{N_2}}\right)} \qquad (6)$$

where,
μ_1 *and* μ_2 are the means for first and second sample respectively [11]. The main limitation of using Microsoft Excel for power estimation for two sample *t* tests with unequal variances using the method described here is that there is no built-in function which can calculate *t*-value with fractional degrees of freedom.

3 Results and discussion

The accuracy method described here was determined by comparing it with the cdf for reported values [12], and the results are shown in Table 2. Figure 4 shows the combination plot of cdf and the noncentral *t* distribution.

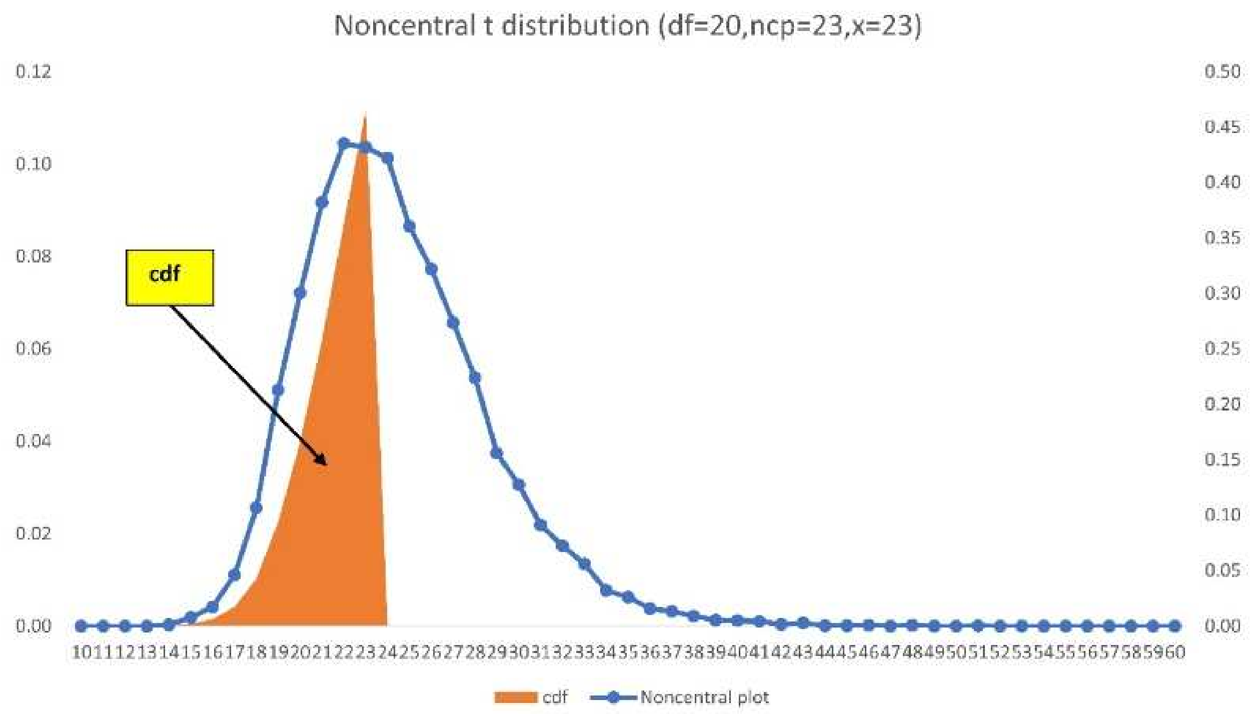

Figure 4 Plot showing cdf computed at 23 for a noncentral t distribution with 20 degrees of freedom, and a noncentrality parameter value of 23

*Table 2 Comparison of reported cdf values vs obtained from the method described in the manuscript. x corresponds to the value at which cdf is computed (*1000 simulations)*

X	ν	δ	Reported [12]	Obtained*
2.34	3	1	0.801888999613917	0.8020529
-4.33	126	-2	1.252846196792878d-2	0.0125672
23	20	23	0.460134400391924	0.4601409
34	20	33	0.532008386378725	0.5319548
39	12	38	0.495868184917805	0.4958221
39	12	39	0.446304024668836	0.4462393
39	200	38	0.666194209961795	0.6664283
40	200	42	0.179292265426085	0.1793178

4 Conclusions

The existing algorithms used to compute cdf are complicated and most users use black box approach to compute cdf using statistical software. The method described in manuscript provides a way of computing cdf which is conceptually easy to understand and can be readily implemented using and any spreadsheet program like Microsoft Excel without the need for any complicated code. This method can also be extended to compute cdf of noncentral χ^2 distribution and noncentral *F* distribution.

References

[1] HARRISON, D.A., BRADY, A.R.: Sample Size and Power Calculations using the Noncentral t-distribution, *The Stata Journal*, Vol. 4, No. 2, pp. 142-153, 2004. https://doi.org/10.1177/1536867x0400400205

[2] RUST, P.F.: *Noncentral t Distribution*, *Encyclopedia of Biostatistics*, Wiley, 2005. https://doi.org/10.1002/0470011815.b2a15097

[3] WITKOVSKY, V.: A Note on Computing Extreme Tail Probabilities of the Noncentral T Distribution with Large Noncentrality Parameter, *Acta Univ. Palacki. Olomuc., Fac. rer. nat., Mathematica*, Vol. 52, No. 2, pp. 131-143, 2013.

[4] Matlab Help Center Noncentral t distribution, [Online], Available: https://www.mathworks.com/help/stats/noncentral-t-distribution.html [03 Sep 2025], 2025.

[5] LIU, X.S.: A Unified Way to Represent and Compute the Cumulative Distribution Function for Noncentral t , F , and χ^2, *Communications in Statistics - Simulation and Computation*, Vol. 42, No. 6, pp. 1433-1436, 2013. https://doi.org/10.1080/03610918.2012.664230

[6] The noncentral t distribution in SAS, [Online], Available: https://blogs.sas.com/content/iml/2022/09/21/noncentral-t.html#:~:text=When%20%CE%B4%20%3E%200%2C%20the%20noncentral,%2C%20CDF%2C%20and%20QUANTILE%20functions [03 Sep 2025], 2022.

[7] UCLA Advanced Research Computing Power analysis for one-sample t-test, [Online], Available: https://stats.oarc.ucla.edu/other/gpower/power-analysis-for-one-sample-t-test/ [03 Sep 2025], 2024.

[8] Calculate multiple results by using a data table, [Online], Available: https://support.microsoft.com/en-us/office/calculate-multiple-results-by-using-a-data-table-e95e2487-6ca6-4413-ad12-77542a5ea50b [03 Sep 2025], 2025.

[9] Introduction to Excel2007 Data Tables and Data Table Exercises, [Online], Available: https://people.wou.edu/~taylors/g302/Excel_2007_Data_Table_Notes.pdf [03 Sep 2025], 2009.

[10] NCSS Chapter 422 Two-Sample T-Tests Assuming Equal Variance, [Online], Available: https://www.ncss.com/wp-content/themes/ncss/pdf/Procedures/PASS/Two-Sample_T-Tests_Assuming_Equal_Variance.pdf [03 Sep 2025], 2025.

[11] NCSS Chapter 424 Two-Sample T-Tests Allowing Unequal Variance, [Online], Available: https://www.ncss.com/wp-content/themes/ncss/pdf/Procedures/PASS/Two-Sample_T-Tests_Allowing_Unequal_Variance.pdf [03 Sep 2025], 2025.

[12] BENTON, D., KRISHNAMOORTHY, K.: Computing discrete mixtures of continuous distributions: noncentral chisquare, noncentral t and the distribution of the square of the sample multiple correlation coefficient, *Computational Statistics & Data Analysis*, Vol. 43, No. 2, pp. 249-267, 2003. https://doi.org/10.1016/S0167-9473(02)00283-9

Review process
Single-blind peer review process.