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Plackett-Burman design

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Abstract: This article focuses on Placket t-Burman designs (P-B plans) for experiments, which represent an efficient tool for experimental designs, especially in investigating the influence of a large number of factors. Unlike traditional designs such as full factorial experiments (FFE) and fractional factorial experiments (FrFE), P-B plans allow for the effective processing of experiments with a higher number of factors without the need for an exponential increase in the number of trials. Although P-B plans do not directly enable the analysis of interactions between factors, they are ideal for exploring and analyzing the main effects of factors. The article examines in detail the advantages and disadvantages of P-B plans, including their ability to provide experimental error estimates and their efficiency in screening studies. The orthogonality and balance of P-B plans are emphasized, and their application in practice is illustrated with an example of a manufacturing process influenced by multiple factors. The approach based on P-B plans is shown to be a significant contribution to the field of experimental design, especially in situations where it is necessary to efficiently evaluate the impact of a large number of factors with a limited number of trials.

1 Introduction

Plackett-Burman designs allow experimental planning, with a focus on handling a multitude of factors without necessitating a corresponding increase in the number of experiments. This unique feature stands in stark contrast to traditional factorial designs, which require a geometric increase in experiments with the addition of each factor, making them less feasible for large-scale studies. Plackett-Burman designs, with their ability to provide a linear increase in the number of experiments, offer a pragmatic solution for preliminary screening in complex experimental setups. By focusing on the main effects and employing a resolution III or IV design, they enable researchers to discern the most influential factors without the convoluted analysis of interactions typical in higher-resolution designs. This approach is not only cost-effective but also time-efficient, making it an invaluable tool in fields where rapid and reliable identification of significant factors is crucial. Plackett-Burman designs have limitations in analysing interactions, which is a drawback. However, this limitation is balanced by their usefulness in the initial stages of experimental design. In these early phases, the main objective is to reduce the list of potential factors for a more detailed analysis. In the realm of experimental design and statistical analysis, the Plackett-Burman design continues to find diverse applications across various scientific disciplines, as evidenced by recent research endeavors. For instance ref. [1], in their study harness the Plackett-Burman design to streamline the experimental process required for evaluating the critical fracture energy in bonded composite joints. This research not only underscores the design's efficacy in reducing experimental redundancy but also critically evaluates its strengths and limitations within the context of mechanical engineering and materials science. Similarly ref. [2], in their publication utilize the Plackett-Burman design followed by a central composite design. This methodological approach aims to refine the process of sulfonamide detection in environmental samples, showcasing the Plackett-Burman design's adaptability in analytical chemistry for optimizing complex extraction processes. Further extending the design's application ref. [3] demonstrate the utilization of Plackett-Burman alongside Central Composite Design. This study focuses on augmenting the production of insecticidal agents from acid-hydrolyzed wastewater, highlighting the design's utility in bioprocess optimization and environmental engineering. Lastly ref. [4] illustrates the application of

Plackett-Burman and Box-Behnken designs in the extraction optimization of phenolics from Seabuckthorn leaves. This study emphasizes the design's role in food science and technology, particularly in developing innovative extraction techniques for valuable natural compounds. These studies collectively underscore the Plackett-Burman design's versatility and efficacy in experimental optimization across a spectrum of research fields, from materials science and environmental engineering to analytical chemistry and food technology. At its core, the article delves into the application and efficacy of Plackett-Burman designs in optimizing experimental frameworks across various scientific domains. Central to this exploration is the design's capacity to efficiently manage a large number of factors with a minimal increase in the number of required experiments, a feature that starkly contrasts with the exponential growth in complexity observed in traditional factorial designs. This article delves into the use of Plackett-Burman designs (P-B plans) in experimental design, particularly in comparison with full factorial experiments (FFE) and fractional factorial experiments (FrFE). P-B plans are presented as suitable for preliminary screening studies, where the main objective is to identify factors with the most significant impact on the phenomenon under study. These plans are characterised as balanced and orthogonal, meaning they provide unbiased estimates of the main effects. The article also discusses the possibilities of creating P-B plans, demonstrating how a specific vector of factors can be used to generate experimental conditions, as illustrated by an example with 7 factors.

2 Methodology

Plackett-Burman plans (P-B plans) for experiments have one significant advantage over full factorial experiments (FFE) and fractional factorial experiments (FrFE), which is their applicability even with a larger number of factors. P-B plans are usually not used for analysing factor interactions but are primarily utilised for in-depth analysis of the main factors. According to ref. [5], a disadvantage of P-B plans is that these plans allow for the evaluation of the impact of up to k = (n-1) factors with n experiments, which is often not necessary in practical use. The remaining columns of a P-B plan can be used for estimating the experiment's error. P-B plans are equivalent to FrFE plans $2_{III}^{(k-p)}$ where *III* is resolution, *k* is number of factors and *p* is number of excluded factors.

In the case of FrFE plans with two levels of factors, the number of required experiments increases geometrically. This results from the formula for the number of experiments for FrFE, where, as mentioned in previous chapters, the number of experiments is equal to $n = 2^{k-p}$, hence for k-p = 1, 2, 3, ... the number of experiments is 2^{k-p} = 2, 4, 6, 8, 16, 32, 64, 128, 256, etc.

For P-B plans, the number of experiments increases arithmetically (resolution level III), and therefore, they can also be used as plans with resolution level IV. In P-B plans, the number of experiments is thus: for k-p = 1, 2, 3, ... the number of experiments is equal to n = 4*k = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, etc., which is a much slower growth compared to FFE and FrFE plans. As can be seen, the number of experiments in Plackett-Burman plans increases by a factor of 4. As [6] states, Plackett-Burman plans are balanced, orthogonal, and allow for the evaluation of effects of a maximum of n_{P-B}= n-1 = 3, 7, 11, 15,... factors or their interactions. PBP work well for linear dependencies of factors and interactions. Non-linear dependencies of factors are usually not included in P-B plans.

Individual rows (experiments) in P-B plans are created using a specific vector n_{P-B}= n-1 factors. For example, using a specific vector n = 7 to create an 8-row P-B plan, where the specific vector is formed (+1, +1, +1, -1, +1, -1, -1).

Now consider a manufacturing process influenced by 7 factors (A, B, C, D, E, F, G). The first 7 rows in column A of the PBP are the specific vector mentioned above. The first row in column B will have the level of the 7th row of column A, i.e., (-1), followed by the first 6 elements of the specific vector from column A. The first row in column C will have the value of the 7th row in column B, i.e., (-1), after which follows the first 6 elements of column B. The first row of column D will have the level of the 7th row of column C, i.e., (+1), after which follows the first 6 rows of column C. The levels of all other columns, up to column 7, are created in a similar manner. The last row, i.e., the 8th row, will have the level (-1) for all columns. The principle of creating P-B plans is evident from Figure 1.

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PBP for 7 factors

Specific vector = (+1, +1, +1, -1, +1, -1, -1)

Trial number	A	B	C	D	E	F	G
1	+1	-1	-1	+1	-1	+1	+1
2	+1	+1	-1	-1	+1	-1	+1
3	+1	+1	+1	-1	-1	+1	-1
4	-1	+1	+1	+1	-1	-1	+1
5	+1	-1	+1	+1	+1	-1	-1
6	-1	+1	-1	+1	+1	+1	-1
7	-1	-1	+1	-1	+1	+1	+1
8	-1	-1	-1	-1	-1	-1	-1

Specific vector

Figure 1 The principle of creating P-B plans

The creation of P-B plans can be easily achieved through the rotation of a specific vector, as shown in Figure 2.

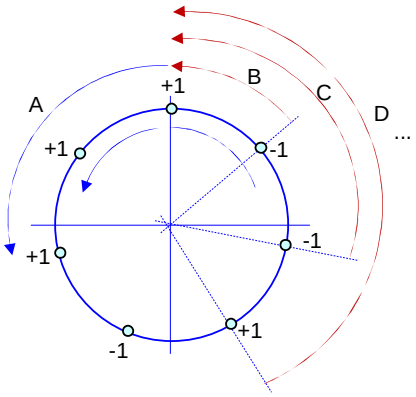

Figure 2 The creation of P-B plans by rotating the specific vector

Table 1 presents the P-B experimental plan for 7 factors, thus comprising 8 rows (trials).

Table 1 P-B plan for 7 factors

Trial number	A	B	C	D	E	F	G
1	+1	-1	-1	+1	-1	+1	+1
2	+1	+1	-1	-1	+1	-1	+1
3	+1	+1	+1	-1	-1	+1	-1
4	-1	+1	+1	+1	-1	-1	+1
5	+1	-1	+1	+1	+1	-1	-1
6	-1	+1	-1	+1	+1	+1	-1
7	**-1**	**-1**	**+1**	**-1**	**+1**	**+1**	+1
8	-1	-1	-1	-1	-1	-1	-1

Examples of specific vectors for P-B plans according to [7]:

n_{P-B} = 4 (-1+1+1)

n_{P-B} = 8 (+1+1+1-1+1-1-1)

n_{P-B} = 12 (+1+1-1+1+1+1-1-1-1+1-1)

n_{P-B} = 16 (+1+1+1+1-1+1-1+1+1-1-1+1-1-1-1)

n_{P-B} = 20 (+1+1-1-1+1+1+1+1-1+1-1+1-1-1-1-1+1+1-1)

In P-B plans, we also carry out factor screening. To display the results, it is appropriate to use a main effects plot. An example of such a graph is shown in Figure 3.

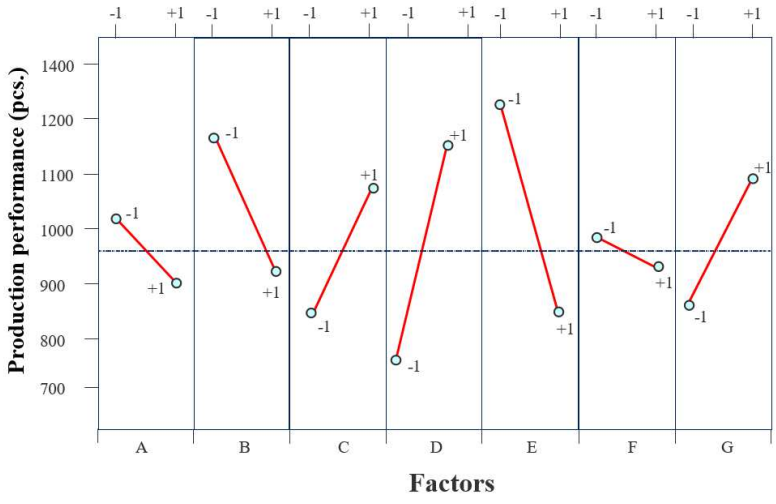

Figure 3 Main effects plot

Simulations, especially those involving complex systems or requiring high computational resources, can be time-consuming and costly [8]. PBPs mitigate this by reducing the number of required experiments to a feasible level, thus saving on both computational resources and time, without compromising the integrity of the findings.

3 Results

Consider an example of a simple production system in which we want to verify the impact of two factors and two levels on its production performance. Details about the factors under investigation are provided in Table 2.

Table 2 Factors and their levels

Factor	Description	Lower level (-)	Upper level (+)
A	Number of maintenance workers	7	12
B	Batch size	200	2800

As seen from Table 3, we have 2 factors, so for FFE, we need to conduct four simulation runs. To ensure at least partial independence, we carry out two simulation experiments, and in each of them, we conduct four simulation runs (thus a total of 8 simulation runs). The results of the experiments are summarised in Table 3.

Table 3 The experiment results

Simulation run (trial)	Factor		Interaction	Production performance	
	A	B	AB	Experiment 1	Experiment 2
1	-1	-1	+1	10	11
2	-1	+1	-1	12	12
3	+1	-1	-1	19	18
4	+1	+1	+1	23	22

Now we can determine the effect of individual factors and their combinations. The effect of a given factor is determined as follows:
A^- = the sum of production performances achieved at the lower level of the factor divided by the number of values.
$A^- = (10 + 11 + 12 + 12) / 4 = 11.25$
$A^+ = (19 + 18 + 23 + 22) / 4 = 20.5$

Then, the resulting effect of factor A will be:
$A = | A^+ - A^- | = | 20.5 - 11.2 | = 9.3$

Similarly, for factor B it will be:
$B^- = 14,5 \qquad B^+ = 17.25$
$B = | B^+ - B^- | = 2.75$

The effect of the combination of factors will be:
$AB^- = 15.25 \qquad AB^+ = 16.5$
$AB = | AB^+ - AB^- | = 1.25$

The greatest effect determines the "main influence." Factor A has the greatest impact, followed by factor B, and the combination of factors AB has the smallest effect. The calculations suggest the optimal combination of factors is A^+B^+. Thus, factors A and B achieve the greatest effect at the level $A^+ B^+$.

4 Discussion and conclusion

The utilisation of Plackett-Burman designs signifies a substantial contribution to the realm of experimental design, particularly in addressing challenges associated with analysing a multitude of factors. This study corroborates their efficacy in preliminary factor screening, enabling researchers to swiftly and efficiently pinpoint the main factors influencing a given phenomenon. A notable advantage of employing Plackett-Burman plans is the elimination of the need for an exponential increase in the number of trials, a common requirement in traditional experimental designs. This aspect is especially beneficial in scenarios where resources and time are limited. However, the inherent limitation of Plackett-Burman designs in not directly addressing factor interactions warrants consideration. While this attribute underscores their suitability for initial stages of experimentation, it may necessitate subsequent, more detailed analysis techniques to fully understand complex factor interrelationships. Therefore, the strategic integration of Plackett-Burman designs with other experimental methods, such as fractional factorial or central composite designs, could provide a more holistic approach to experimental investigation. The practical implications of this study extend across various scientific domains, demonstrating the versatility of Plackett-Burman designs. Whether in optimising manufacturing processes, enhancing product quality, or streamlining research methodologies, the application of these designs can lead to significant advancements and innovations. Furthermore, the ability to conduct efficient screening studies with Plackett-Burman plans empowers researchers and industry professionals alike to make informed decisions, ultimately contributing to the enhancement of experimental design practices. In conclusion, Plackett-Burman designs represent a valuable strategy for managing the complexity inherent in experimental studies involving a large number of factors.

Through the efficient identification of main effects, these designs facilitate a streamlined approach to experimental planning, reducing both time and resource commitments. Despite limitations regarding interaction analysis, the strategic application of Plackett-Burman designs, in conjunction with other analytical methods, offers a comprehensive framework for experimental exploration.

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